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Effects of the right-handed neutrinos on $\Delta S = 2$ and $\Delta B = 2$ processes in supersymmetric $\mathrm{SU}(5)$ model

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Abstract: We discuss an extra source of CP and flavor violations in supersymmetric SU(5) grand unified model with the right-handed neutrinos. In such a model, the right-handed down-type squarks $\tilde{d}_R$ interact with the right-handed neutrinos above the GUT scale, and the renormalization group effect can generate sizable off-diagonal elements in the mass matrix of $\tilde{d}_R$. Because of new Yukawa phases which exist in the SU(5) model, these off-diagonal elements have, in general, large CP violating phases. The renormalization group induced off-diagonal elements affect the K and B decays. In particular, in this model, supersymmetric contribution to the ϵ_K parameter can be as large as the currently measured experimental value, and hence the effect might be seen as an anomaly in the on-going test of the Cabibbo-Kobayashi-Maskawa triangle.

Keywords: Supersymmetric Standard Model, GUT, Neutrino Physics, Kaon Physics.

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In particle physics, one of the greatest excitement in the last decade is the discovery of the evidence for the neutrino oscillations [1]. In particular, the anomalies in the atmospheric and solar neutrino fluxes suggest neutrino masses much smaller than those of the quarks and charged leptons.

Such small neutrino masses are beautifully explained by the see-saw mechanism with the right-handed neutrinos [2, 3]. The see-saw mechanism gives the (left-handed) neutrino masses of the form $m_{\nu_L} \sim m_{\nu_\mathrm{D}}^2/M_{\nu_R}$, where m_{ν_D} and M_{ν_R} are Dirac and Majorana masses for neutrinos, respectively. The Dirac mass m_{ν_D} is from the Yukawa interaction with the electroweak Higgs boson, and is of order the electroweak scale M_weak (or smaller). The Majorana mass M_{ν_R} is, however, not related to the electroweak symmetry breaking and can be much larger than M_weak. Adopting $M_{\nu_R} \gg M_\mathrm{weak}$, $m_{\nu_L} \ll M_\mathrm{weak}$ is realized. For example, assuming $m_{\nu_\mathrm{D}} \sim M_\mathrm{weak}$, the atospheric neutrino anomaly predicts $M_{\nu_R} \sim 10^{14} \div 10^{15}$ GeV with relevant mixing parameters.

This fact suggests the validity of the field theoretical description up to a scale much higher than the electroweak scale. If so, the quadratic divergence in the Higgs boson mass parameter badly destabilize the electroweak scale unless some mechanism protects the stability. A natural solution to this problem is to introduce supersymmetry (SUSY); in SUSY models, quadratic divergences are cancelled between bosonic and fermionic loops and hence the electroweak scale becomes stable against radiative corrections. SUSY standard model also suggests an interesting new physics at a high scale, i.e., the grand unified theory (GUT). With the renormalization group (RG) equation based on the minimal SUSY standard model (MSSM), all the gauge coupling constants meet at $M_\mathrm{GUT} \simeq 2 \times 10^{16}$ GeV, while the gauge coupling unification is not realized in the (non-supersymmetric) standard model [4, 5].

Therefore, SUSY GUT with the right-handed neutrinos is a well-motivated extension of the standard model. In this paper, we consider $\Delta S = 2$ and $\Delta B = 2$ processes in the SU(5) GUT with the right-handed neutrinos. Before the Super-Kamiokande experiment [6], similar effects were considered without taking into account the large mixing in the neutrino sector [7]. Now, we have better insights into the neutrino mass and mixing parameters. In particular, the atmospheric neutrino flux measured by the Super-Kamiokande experiment strongly favors a large 2-3 mixing in the neutrino sector [6]. Furthermore, with the on-going B factories, the mixings in the quark sector, in particular, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, will be well constrained. In such a circumstance, it is appropriate to study implications of the right-handed neutrinos to the $\Delta S = 2$ and $\Delta B = 2$ processes within the current experimental situation.

In grand unified models, the right-handed down type squarks $\tilde{d}_R$ couple to the right-handed neutrinos above the GUT scale, and sizable flavor violations are possible in the $\tilde{d}_R$ sector. In particular, the RG effect may generate off-diagonal elements in the soft SUSY breaking mass matrix of $\tilde{d}_R$. (Notice that the flavor violation in the $\tilde{d}_R$ sector is highly suppressed in the minimal supergravity model without the right-handed neutrinos.) Furthermore, in grand unified models, there are new physical CP violating phases in the colored Higgs vertices, which do not affect the low energy Yukawa interactions. These may affect CP and flavor violating processes in K and B system. Indeed, as we will see, SUSY contribution to the ϵ_K parameter can be as large as the experimental value even with a relatively heavy squark mass $m_{\tilde{q}} \sim 1$ TeV, and it might cause an anomaly in the on-going test of the CKM triangle.

We start by introducing the lagrangian for the SUSY SU(5) model with the right-handed neutrinos. Denoting $\mathbf{10}$, $\bar{\mathbf{5}}$, and singlet fields of SU(5) as Ψ_i, Φ_i, and N_i, respectively, the superpotential is given by[1]

$$W_{\rm GUT} = \frac{1}{8}\Psi_i \left[Y_U\right]_{ij} \Psi_j H + \Psi_i \left[Y_D\right]_{ij} \Phi_j \bar{H} + N_i \left[Y_N\right]_{ij} \Phi_j H + \frac{1}{2} N_i \left[M_N\right]_{ij} N_j \,, \qquad (1)$$

where $\bar{H}$ and H are Higgs fields which are in $\bar{\mathbf{5}}$ and $\mathbf{5}$ representations, respectively, and i and j are generation indices which run from 1 to 3. (We omit the SU(5) indices for the simplicity of the notation.) Here, M_N is the Majorana mass matrix of the right-handed neutrinos. In order to make our points clearer, we adopt the simplest Majorana mass matrix, that is, the universal structure of M_N:

$$\left[M_N\right]_{ij} = M_{\nu_R} \delta_{ij} \,. \qquad (2)$$

The following discussions are qualitatively unaffected by this assumption, although numerical results depend on the structure of M_N. In eq. (1), Y's are 3×3 Yukawa matrices; in general, Y_U is a general complex symmetric matrix, while Y_D and Y_N are general complex matrices. By choosing a particular basis, we can eliminate unphysical phases and angles. In this paper, we choose the basis of Ψ_i, Φ_i, and N_i such that the Yukawa matrices at the GUT scale become

$$Y_U(M_{\rm GUT}) = V_Q^T \hat{\Theta}_Q \hat{Y}_U V_Q \,, \qquad Y_D(M_{\rm GUT}) = \hat{Y}_D \,, \qquad Y_N(M_{\rm GUT}) = \hat{Y}_N V_L \hat{\Theta}_L \,, \quad (3)$$

where $\hat{Y}$'s are real diagonal matrices:

$$\hat{Y}_U = {\rm diag}(y_{u_1}, y_{u_2}, y_{u_3}) \,, \qquad \hat{Y}_D = {\rm diag}(y_{d_1}, y_{d_2}, y_{d_3}) \,, \qquad \hat{Y}_N = {\rm diag}(y_{\nu_1}, y_{\nu_2}, y_{\nu_3}) \,, \qquad (4)$$

while $\hat{\Theta}$'s are diagonal phase matrices:

$$\hat{\Theta}_Q = {\rm diag}\left(e^{i\phi_1^{(Q)}}, e^{i\phi_2^{(Q)}}, e^{i\phi_3^{(Q)}}\right), \qquad \hat{\Theta}_L = {\rm diag}\left(e^{i\phi_1^{(L)}}, e^{i\phi_2^{(L)}}, e^{i\phi_3^{(L)}}\right), \qquad (5)$$

where phases obey the constraints $\phi_1^{(Q)} + \phi_2^{(Q)} + \phi_3^{(Q)} = 0$ and $\phi_1^{(L)} + \phi_2^{(L)} + \phi_3^{(L)} = 0$. Furthermore, V_Q and V_L are unitary mixing matrices and are parameterized as [8]

$$V_Q = \begin{pmatrix} c_{12}^{(Q)} c_{13}^{(Q)} & s_{12}^{(Q)} c_{13}^{(Q)} & s_{13}^{(Q)} e^{-i\delta_{13}^{(Q)}} \\ -s_{12}^{(Q)} c_{23}^{(Q)} - c_{12}^{(Q)} s_{23}^{(Q)} s_{13}^{(Q)} e^{i\delta_{13}^{(Q)}} & c_{12}^{(Q)} c_{23}^{(Q)} - s_{12}^{(Q)} s_{23}^{(Q)} s_{13}^{(Q)} e^{i\delta_{13}^{(Q)}} & s_{23}^{(Q)} c_{13}^{(Q)} \\ s_{12}^{(Q)} s_{23}^{(Q)} - c_{12}^{(Q)} c_{23}^{(Q)} s_{13}^{(Q)} e^{i\delta_{13}^{(Q)}} & -c_{12}^{(Q)} s_{23}^{(Q)} - s_{12}^{(Q)} c_{23}^{(Q)} s_{13}^{(Q)} e^{i\delta_{13}^{(Q)}} & c_{23}^{(Q)} c_{13}^{(Q)} \end{pmatrix}, \qquad (6)$$

where $c_{ij}^{(Q)} = \cos\theta_{ij}^{(Q)}$ and $s_{ij}^{(Q)} = \sin\theta_{ij}^{(Q)}$, and $V_L = V_Q|_{Q\to L}$.

[1] We assume that other Yukawa couplings (in particular, those of the SU(5) symmetry breaking fields) are small. We neglect their effects in this paper.

$W_{\rm GUT}$ can be also expressed by using the standard model fields. By properly relating the GUT fields with the standard model fields, the phases $\phi^{(Q)}$ and $\phi^{(L)}$ are eliminated from the interactions among the light fields. Indeed, let us embed the standard model fields into the GUT fields as

$$\Psi_i \simeq \{Q, V_Q^\dagger \hat{\Theta}_Q^\dagger \bar{U}, \hat{\Theta}_L \bar{E}\}_i\,, \qquad \Phi_i \simeq \{\bar{D}, \hat{\Theta}_L^\dagger L\}_i, \tag{7}$$

where $Q_i(\mathbf{3},\mathbf{2})_{1/6}$, $\bar{U}_i(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$, $\bar{D}_i(\bar{\mathbf{3}},\mathbf{1})_{1/3}$, $L_i(\mathbf{1},\mathbf{2})_{1/2}$, and $\bar{E}_i(\mathbf{1},\mathbf{1})_1$ are quarks and leptons in i-th generation with the $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ gauge quantum numbers as indicated. With this embedding, $W_{\rm GUT}$ at the GUT scale becomes

$$\begin{aligned} W_{\rm GUT} = W_{\rm SSM} &- \frac{1}{2} Q_i \left[V_Q^T \hat{\Theta}_Q \hat{Y}_U V_Q\right]_{ij} Q_j H_C + \bar{E}_i \left[\hat{\Theta}_L \hat{Y}_U\right]_{ij} \bar{U}_j H_C + \\ &+ \bar{U}_i \left[\hat{\Theta}_Q^\dagger V_Q^* \hat{Y}_D\right]_{ij} \bar{D}_j \bar{H}_C - Q_i \left[\hat{Y}_D \hat{\Theta}_L^\dagger\right]_{ij} L_j \bar{H}_C + N_i \left[\hat{Y}_N V_L \hat{\Theta}_L\right]_{ij} \bar{D}_j H_C\,, \end{aligned} \tag{8}$$

where H_C and $\bar{H}_C$ are colored Higgses, and the low energy lagrangian is given by

$$\begin{aligned} W_{\rm SSM} = Q_i \left[V_Q^T \hat{Y}_U\right]_{ij} \bar{U}_j H_u + Q_i \left[\hat{Y}_D\right]_{ij} \bar{D}_j H_d + \bar{E}_i \left[\hat{Y}_E\right]_{ij} L_j H_d + \\ + N_i \left[\hat{Y}_N V_L\right]_{ij} L_j H_u + \frac{1}{2} M_{\nu_R} N_i N_i\,, \end{aligned} \tag{9}$$

with H_u and H_d being the up- and down-type Higgses, respectively. Simple SU(5) GUT predicts $\hat{Y}_D = \hat{Y}_E$, but this relation breaks down for the first and second generations. Some non-trivial flavor physics is necessary to have a realistic down-type quark and charged lepton Yukawa matrices. Such a new physics may provide a new source of flavor and CP violations [9, 10], but we do not consider such an effect in this paper.

In the basis given in eq. (9), Yukawa matrices for the down-type quarks and charged leptons are diagonal. Strictly speaking, the RG effect mixes the flavors and their diagonalities are not preserved at lower scale. Such a flavor mixing is, however, rather a minor effect for the Yukawa matrices and the down-type quarks and charged leptons in eq. (9) correspond to the mass eigenstates quite accurately. Therefore, this basis is useful to understand the qualitative features of the K and B physics. So, we first discuss the flavor violation of the model using eq. (9) neglecting the RG effect for the Yukawa matrices below the GUT scale.[2]

With the superpotential (9), the left-handed neutrino mass matrix is given by

$$[m_{\nu_L}]_{ij} = \frac{\langle H_u \rangle^2}{M_{\nu_R}} \left[V_L^T \hat{Y}_N^2 V_L\right]_{ij} = \frac{v^2 \sin^2\beta}{2 M_{\nu_R}} \left[V_L^T \hat{Y}_N^2 V_L\right]_{ij}\,, \tag{10}$$

[2]Notice that, in our numerical discussion later, all the flavor mixing effects are included. In particular, mass eigenstates of the quarks and charged leptons will be defined using the Yukawa matrices given at the electroweak scale.

where $v \simeq 246\,\mathrm{GeV}$, and β parameterizes the relative size of two Higgs vacuum expectation values: $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$. Then, V_L provides the mixing matrix of the lepton sector. On the contrary, as seen in eq. (9), V_Q is the CKM matrix: $V_Q \simeq V_{\mathrm{CKM}}$.

At the tree level, effects of the heavy particles, like the colored Higgses and right-handed neutrinos, are suppressed by inverse powers of their masses. As a result, we can mostly completely neglect the interaction with such heavy particles, although there are a few important processes like the neutrino oscillation and the proton decay. The phases $\phi^{(Q)}$ and $\phi^{(L)}$ do not affect the Yukawa interactions among the standard model fields and only appear in the colored Higgs vertices. Therefore, their effects on low energy physics are rather indirect, and those phases are not determined from the Yukawa interactions of the quarks and leptons with the electroweak Higgses.

At the loop level, however, heavy particles may affect the low energy quantities through the RG effect [11, 12, 13, 14]. In particular, flavor changing operators may be significantly affected. For example, the neutrino Yukawa interactions may induce off-diagonal elements in the soft SUSY breaking mass matrix of left-handed sleptons. In addition, even if there is no neutrino Yukawa interactions, lepton flavor numbers are violated in the grand unified models by the up-type Yukawa interaction, and hence we may expect non-vanishing off-diagonal elements in the mass matrix of the right-handed sleptons. Since the lepton flavor numbers are preserved in the standard model, these effects result in very drastic signals like $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in SU(5) [14, 15, 16] and SO(10) [15, 17, 9] models as well as in models with the right-handed neutrinos [12, 18, 19, 20]. Furthermore, hadronic flavor and CP violations in grand unified models were considered in ref. [7] without taking into account the large mixing in the neutrino sector. The flavor mixing below the GUT scale through the CKM matrix also affects the $\Delta S = 2$ and $\Delta B = 2$ processes [21, 22, 23, 24]. Here, we consider CP violation and flavor mixing in the down sector induced by the neutrino Yukawa matrix. As seen in eq. (8), $\bar{D}$ interacts with the right-handed neutrinos above the GUT scale, and its soft SUSY breaking scalar mass matrix is affected by the neutrino Yukawa interactions. Because of possible flavor and CP violating parameters in V_L and $\hat{\Theta}_L$, this can be important. Similar effect was also considered in ref. [20] for the $b \to s\gamma$ process, for which the effect was insignificant. Here, we will show that a large effect is possible in particular in the CP violation in the K decay. Furthermore, ref. [20] did not take into account the physical phases $\phi^{(L)}$ which play a very important role in calculating the SUSY contribution to the ϵ_K parameter.

Now, we are at the point to discuss the RG effect and to estimate the off-diagonal elements in the down-type squark mass matrix. The RG effect is not calculable unless the boundary condition of the SUSY breaking parameters are given. In order to estimate the importance of this effect, we use the minimal supergravity boundary condition. Then, the SUSY breaking parameters are parameterized by the following three parameters:[3] the universal scalar mass m_0, the universal A-parameter a_0 which is the ratio of SUSY breaking trilinear scalar interactions to corresponding Yukawa couplings, and the SU(5) gaugino mass m_{G5}. We assume that the boundary condition is given at the reduced Planck scale $M_* \simeq 2.4 \times 10^{18}$ GeV.

Once the boundary condition is given, the MSSM lagrangian at the electroweak scale is obtained by running all the parameters down from the reduced Planck scale to the electroweak scale with RG equation. For the RG analysis, relevant effective field theory describing the energy scale μ is MSSM (without the right-handed neutrinos) for $M_{\rm weak} \leq \mu \leq M_{\nu_R}$, SUSY standard model with the right-handed neutrinos for $M_{\nu_R} \leq \mu \leq M_{\rm GUT}$, and SUSY SU(5) GUT for $M_{\rm GUT} \leq \mu \leq M_*$. We use the minimal SUSY SU(5) GUT where the SU(5) symmetry is broken by **24** representation of SU(5).

Before showing the results of the numerical calculation, let us discuss the behavior of the solution to the RG equation using a simple approximation. With the one-iteration approximation, off-diagonal elements in the soft SUSY breaking mass of $\tilde{d}_R$ is given by

$$\begin{aligned} [m^2_{\tilde{d}_R}]_{ij} &\simeq -\frac{1}{8\pi^2} \left[Y_N^\dagger Y_N \right]_{ij} (3m_0^2 + a_0^2) \log \frac{M_*}{M_{\rm GUT}} \\ &\simeq -\frac{1}{8\pi^2} e^{-i(\phi_i^{(L)} - \phi_j^{(L)})} y^2_{\nu_k} [V_L^*]_{ki} [V_L]_{kj} (3m_0^2 + a_0^2) \log \frac{M_*}{M_{\rm GUT}} . \end{aligned} \tag{11}$$

This expression suggests two important consequences of the SU(5) model with the right-handed neutrinos. First, if neutrinos have a large Yukawa coupling and mixing, off-diagonal elements in $[m^2_{\tilde{d}_R}]_{ij}$ are generated. Since the atmospheric and solar neutrino anomalies require non-vanishing mixing parameters in V_L, this effect may generate sizable off-diagonal elements in $[m^2_{\tilde{d}_R}]$. Notice that, below the GUT scale, $\tilde{d}_R$ have very weak flavor violating Yukawa interactions. As a result, if the minimal supergravity boundary condition is given at the GUT scale, we see much smaller flavor violation in $[m^2_{\tilde{d}_R}]$. The second point is on the phase of the RG induced off-diagonal elements. Because the phases $\phi^{(L)}$ are free parameters, the RG induced off-diagonal elements (11) may have arbitrary phases and large CP violating phases in the mass matrix of $\tilde{d}_R$ are possible.

As can be understood in eq. (11), $[m^2_{\tilde{d}_R}]_{ij}$ depends on the structure of the neutrino mixing matrix V_L. In our analysis, we use the mixing angles and neutrino masses suggested by the atmospheric neutrino data, and also by the large angle or small

[3]There are extra important parameters. In particular, in order to have a viable electroweak symmetry breaking, so-called μ- and B_μ-parameters are necessary. However, they do not affect the running of other parameters, so they do not have to be specified in our RG analysis. We assume they have correct values required from the radiative electroweak breaking.

angle MSW solution to the solar neutrino problem [25].[4] For the large angle MSW case, we use

$$m_\nu \simeq (0, 0.004\,\text{eV}, 0.03\,\text{eV})\,, \qquad V_L \simeq \begin{pmatrix} 0.91 & -0.30 & 0.30 \\ 0.42 & 0.64 & -0.64 \\ 0 & 0.71 & 0.70 \end{pmatrix}, \tag{12}$$

and for the small angle case, we use

$$m_\nu \simeq (0, 0.003\,\text{eV}, 0.03\,\text{eV})\,, \qquad V_L \simeq \begin{pmatrix} 1.0 & -0.028 & 0.028 \\ 0.040 & 0.70 & -0.71 \\ 0 & 0.71 & 0.70 \end{pmatrix}. \tag{13}$$

Here, we assume that the neutrino masses are hierarchical, and that the lightest neutrino mass is negligibly small. The 3-1 element of V_L is known to be relatively small: $|[V_L]_{31}|^2 \leq 0.05$ [25] from the CHOOZ experiment [26]. We simply assume $[V_L]_{31}$ be small enough to be neglected, and we take $[V_L]_{31} = 0$. We will discuss the implication of $[V_L]_{31} \neq 0$ later. In calculating the neutrino Yukawa matrix, we take M_{ν_R} as a free parameter, and the neutrino Yukawa matrix is determined as a function of M_{ν_R} using eq. (10).

In order to discuss the flavor violating effects, it is convenient to define $\Delta_{ij}^{(R)}$, which is the off-diagonal elements in $[m^2_{\tilde{d}_R}]_{ij}$ normalized by the squark mass:

$$\Delta_{ij}^{(R)} \equiv \frac{[m^2_{\tilde{d}_R}]_{ij}}{m^2_{\tilde{q}}} \equiv \frac{[m^2_{\tilde{d}_R}]_{ij}}{[m^2_{\tilde{d}_R}]_{11}}, \tag{14}$$

where all the quantities are evaluated at the electroweak scale. Notice that, in our case, all the squark masses are almost degenerate except those for the stops. We use the first generation right-handed squark mass as a representative value.

In figure 1, we plot several $|\Delta_{ij}^{(R)}|$ as a function of M_{ν_R}. Here, we take $a_0 = 0$, $\tan\beta = 3$, and m_{G5} and m_0 are chosen so that $m_{\tilde{q}} = 1\,\text{TeV}$ and $m_{\tilde{W}} = 150\,\text{GeV}$ (which gives the gluino mass of $m_{\tilde{G}} \simeq 520\,\text{GeV}$). As one can see, for larger M_{ν_R}, off-diagonal ele-

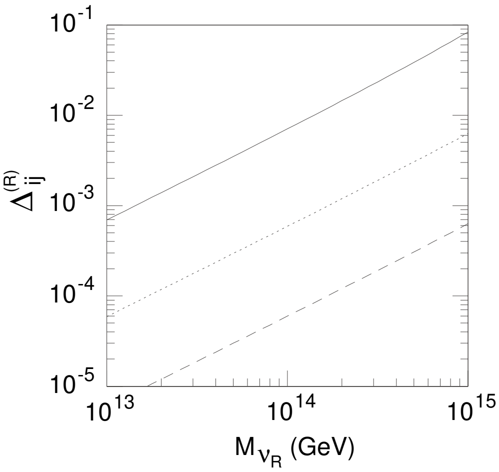

Figure 1: The flavor violation parameters defined in eq. (14); $|\Delta_{23}^{(R)}|$ in the large angle MSW case (solid), $|\Delta_{12}^{(R)}|$ in the large angle MSW case (dotted), and $|\Delta_{12}^{(R)}|$ in the small angle MSW case (dashed). We take $a_0 = 0$, $\tan\beta = 3$, and m_{G5} and m_0 are chosen so that $m_{\tilde{q}} = 1\,\text{TeV}$ and $m_{\tilde{W}} = 150\,\text{GeV}$.

[4] We can also consider the case of the vacuum oscillation solution to the solar neutrino problem. The vacuum oscillation solution requires very light second generation neutrino ($m_{\nu_2} \sim O(10^{-5}\,\text{eV})$). With our simple set up, y_{ν_2} cannot be large once the perturbativity of y_{ν_3} is imposed up to M_*, since we assume the universal structure of M_N. Consequently, the RG effect is suppressed except for the 2-3 mixing. If we consider general neutrino mass matrix, however, this may not be true. For example, non-universal M_N or non-vanishing $[V_L]_{31}$ would easily enhance the RG effect.

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ments are more enhanced. This is because the neutrino Yukawa coupling is proportional to $M_{\nu_R}^{1/2}$ for fixed neutrino mass. $\Delta_{23}^{(R)}$ is dominated by the 2-3 mixing in the neutrino sector, and hence both the large and small mixing cases give almost the same $\Delta_{23}^{(R)}$. (Therefore, we plot only $\Delta_{23}^{(R)}$ in the large mixing case.) On the contrary, with the choice of V_L given in eqs. (12) and (13), $[V_L]_{31} = 0$, and hence both $\Delta_{12}^{(R)}$ and $\Delta_{13}^{(R)}$ are from the term proportional to $y_{\nu_2}^2$ in eq. (11). As a result, these quantities are more enhanced for the large angle MSW case because of the larger $|[V_L]_{21}|$. Furthermore, since $|[V_L]_{22}| \simeq |[V_L]_{23}|$ in the mixing matrices we use, $|\Delta_{12}^{(R)}| \simeq |\Delta_{13}^{(R)}|$ is realized both in the large and small mixing cases. If $[V_L]_{31} \sim O(10^{-2})$, however, $y_{\nu_3}^2$-term may give comparable contributions. We also found that $|\Delta_{ij}^{(R)}|$ is almost independent of the phases $\phi^{(Q)}$ and $\phi^{(L)}$. As suggested from eq. (11), however, the phase of $\Delta_{ij}^{(R)}$ is sensitive to $\phi^{(L)}$; the phase of $\Delta_{ij}^{(R)}$ agrees with $\phi_i^{(L)} - \phi_j^{(L)}$ very accurately. Therefore, $\Delta_{ij}^{(R)}$ may have an arbitrary phase and can be a new source of CP violation.

The most stringent constraints on the off-diagonal elements in the squark mass matrices are from the $\Delta S = 2$ and $\Delta B = 2$ processes [27], so we discuss them in this paper. In particular, $|\Delta_{12}^{(R)}| \sim O(10^{-4})$ realized with $M_{\nu_R} \sim 10^{14} \div 10^{15}\,\mathrm{GeV}$ may generate the ϵ_K parameter as large as the experimental value [27]. In order to discuss the SUSY contribution to the flavor and CP violations (in particular, to the ϵ_K parameter), we calculate the effective hamiltonian contributing to $\Delta S = 2$ and $\Delta B = 2$ processes:[5]

$$\begin{aligned}\mathcal{H}_{\rm eff} = {} & [C_{LL}]_{ij} \left(\bar{d}_i^a \gamma_\mu P_L d_j^a\right)\left(\bar{d}_i^b \gamma_\mu P_L d_j^b\right) + [C_{RR}]_{ij} \left(\bar{d}_i^a \gamma_\mu P_R d_j^a\right)\left(\bar{d}_i^b \gamma_\mu P_R d_j^b\right) + \\ & + \left[C_{LR}^{(1)}\right]_{ij} \left(\bar{d}_i^a P_L d_j^a\right)\left(\bar{d}_i^b P_R d_j^b\right) + \left[C_{LR}^{(2)}\right]_{ij} \left(\bar{d}_i^a P_L d_j^b\right)\left(\bar{d}_i^b P_R d_j^a\right), \end{aligned} \tag{15}$$

where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$, d_i^a is the down-type quark in i-th generation with a being $\mathrm{SU}(3)_C$ index. Based on the operator structure, we call the first and second operators LL and RR operators, respectively, while the third and fourth ones LR operators. In our analysis, we only include the dominant contributions from the gluino-squark loops, and use mass-insertion approximation. The SUSY contributions to the coefficients are found in ref. [27], and are given by

$$\begin{aligned} [C_{LL}]_{ij} &= -\frac{\alpha_s}{36 m_{\tilde{q}}^2} \left(\Delta_{ij}^{(L)}\right)^2 \left[4x f_6(x) + 11 \tilde{f}_6(x)\right] \\ [C_{RR}]_{ij} &= -\frac{\alpha_s}{36 m_{\tilde{q}}^2} \left(\Delta_{ij}^{(R)}\right)^2 \left[4x f_6(x) + 11 \tilde{f}_6(x)\right] \\ \left[C_{LR}^{(1)}\right]_{ij} &= -\frac{\alpha_s}{3 m_{\tilde{q}}^2} \Delta_{ij}^{(L)} \Delta_{ij}^{(R)} \left[7x f_6(x) - \tilde{f}_6(x)\right] \\ \left[C_{LR}^{(2)}\right]_{ij} &= -\frac{\alpha_s}{9 m_{\tilde{q}}^2} \Delta_{ij}^{(L)} \Delta_{ij}^{(R)} \left[x f_6(x) + 5 \tilde{f}_6(x)\right], \end{aligned} \tag{16}$$

[5] In eq. (15), possible operators arising from the left-right squark mixing are omitted, although those are included in our numerical calculation. We checked their contributions are negligible relative to the dominant contributions from $\Delta_{ij}^{(R)}$ (and $\Delta_{ij}^{(L)}$).

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where $x = m_{\tilde{G}}^2/m_{\tilde{q}}^2$, and

$$f_6(x) = \frac{6(1+3x)\log x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5}$$
$$\tilde{f}_6(x) = \frac{6x(1+x)\log x - x^3 - 9x^2 + 9x + 1}{6(x-1)^5}\,. \tag{17}$$

Furthermore, $\Delta_{ij}^{(L)}$ parameterizes the flavor violation in the left-handed down-type squarks $\tilde{d}_L$:

$$\Delta_{ij}^{(L)} \equiv \frac{[m_{\tilde{d}_L}^2]_{ji}}{m_{\tilde{q}}^2}\,. \tag{18}$$

With the minimal supergravity boundary condition, $[m_{\tilde{d}_L}^2]_{ij}$ is generated by the large top Yukawa coupling y_t and the CKM matrix. Approximately,

$$[m_{\tilde{d}_L}^2]_{ij} \simeq -\frac{1}{8\pi^2} y_t^2\,[V_{\mathrm{CKM}}^*]_{3i}\,[V_{\mathrm{CKM}}]_{3j}\,(3m_0^2 + a_0^2)\left(3\log\frac{M_*}{M_{\mathrm{GUT}}} + \log\frac{M_{\mathrm{GUT}}}{M_{\mathrm{weak}}}\right), \tag{19}$$

and numerically, for example, $|\Delta_{12}^{(R)}|$ is $O(10^{-4})$.

With the effective hamiltonian (15), we first calculate the SUSY contribution to ϵ_K:

$$\epsilon_K = \frac{e^{i\pi/4}\,\mathrm{Im}\langle K|\mathcal{H}_{\mathrm{eff}}|\bar{K}\rangle}{2\sqrt{2} m_K \Delta m_K}\,, \tag{20}$$

where m_K and Δm_K are the K-meson mass and the K_L-K_S mass difference, respectively. QCD corrections below the electroweak scale are neglected. The matrix element of the effective hamiltonian is calculated by using the vacuum insertion approximation:

$$\begin{aligned}\langle K|\mathcal{H}_{\mathrm{eff}}|\bar{K}\rangle = {} & \frac{2}{3}\left([C_{LL}]_{12} + [C_{RR}]_{12}\right) m_K^2 f_K^2 + \\ & + \left[C_{LR}^{(1)}\right]_{12}\left(\frac{1}{2}\frac{m_K^2}{(m_s+m_d)^2} + \frac{1}{12}\right) m_K^2 f_K^2 + \\ & + \left[C_{LR}^{(2)}\right]_{12}\left(\frac{1}{6}\frac{m_K^2}{(m_s+m_d)^2} + \frac{1}{4}\right) m_K^2 f_K^2\,,\end{aligned} \tag{21}$$

where $f_K \simeq 160$ MeV is the K-meson decay constant, and m_s and m_d are the strange and down quark masses, respectively.

The new flavor and CP violating effects in the $\tilde{d}_R$ sector have an important implication to the calculation of ϵ_K, since the matrix element of the LR-type operators are enhanced by the factor $(m_K/m_s)^2 \simeq 10$, as seen in eq. (21). As a result, if all the coefficients of the LL, RR, and LR operators are comparable, the LR ones dominate the contribution to the ϵ_K parameter. In the SUSY SU(5) model with the right-handed neutrinos, this is the case. However, if there is no right-handed neutrinos, or if the running above the GUT scale is not included, $\Delta_{12}^{(R)}$ is much more suppressed and the LL operator gives the dominant effect. Then, the SUSY contribution to ϵ_K becomes much smaller [23, 24].

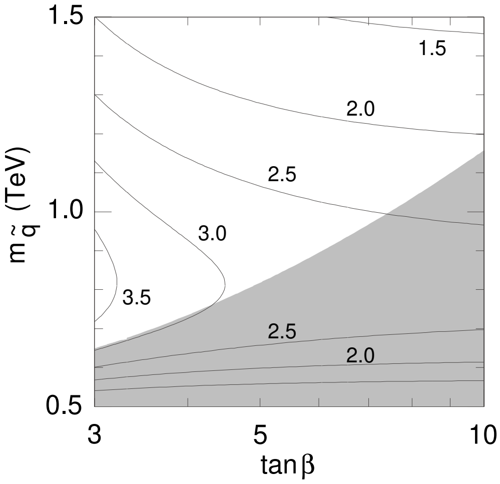

Figure 2: Contours of constant $|\epsilon_K^{(\mathrm{SUSY})}|$ (in units of 10^{-3}) on $\tan\beta$ vs. $m_{\tilde{q}}$ plane for the large angle MSW case (12). We take $m_{\tilde{W}} = 150\,\mathrm{GeV}$, $M_{\nu_R} = 2\times 10^{14}\,\mathrm{GeV}$, $a_0 = 0$, and the GUT phases are chosen so that $|\epsilon_K^{(\mathrm{SUSY})}|$ is maximized. The shaded region corresponds to $Br(\mu\to e\gamma) \geq 4.9\times 10^{-11}$.

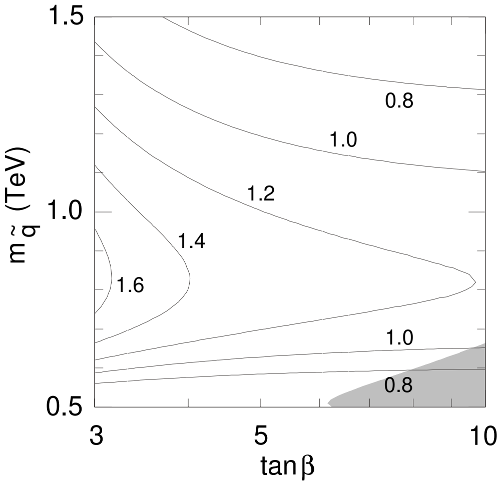

Figure 3: Same as figure 2 except the neutrino mass matrix suggested by the small angle MSW case (13), and $M_{\nu_R} = 10^{15}\,\mathrm{GeV}$.

We solve the RG equation numerically and obtain the electroweak scale values of the MSSM parameters. Then, we calculate $\epsilon_K^{(\mathrm{SUSY})}$, the SUSY contribution to the ϵ_K parameter. In figures 2 and 3, we show $|\epsilon_K^{(\mathrm{SUSY})}|$ on $\tan\beta$ vs. $m_{\tilde{q}}$ plane. Here, we take $m_{\tilde{W}} = 150\,\mathrm{GeV}$, and consider both the large and small angle MSW cases. The result depends on the phase $\phi_2^{(L)} - \phi_1^{(L)}$; $\epsilon_K^{(\mathrm{SUSY})}$ is approximately proportional to $\sin(\phi_2^{(L)} - \phi_1^{(L)} + \arg(\Delta_{12}^{(L)}))$. We choose $\phi_2^{(L)} - \phi_1^{(L)}$ which maximizes $|\epsilon_K^{(\mathrm{SUSY})}|$. The RG effect also generates off-diagonal elements in the slepton mass matrix, which induce various lepton flavor violations. In particular, in our case, rate for the $\mu\to e\gamma$ process may become significantly large. In figures 2 and 3, we shaded the region with $Br(\mu\to e\gamma)\geq 4.9\times 10^{-11}$, which is already excluded by the negative search for $\mu\to e\gamma$ [8]. The $\mu\to e\gamma$ process is enhanced for large $\tan\beta$ [18, 9, 19]. However, large amount of parameter space is still allowed, in particular, in the low $\tan\beta$ region. As one can see, the SUSY contribution to ϵ_K can be $O(10\%)$ of the experimentally measured value ($\epsilon_K^{(\mathrm{exp})} \simeq 2.3\times 10^{-3}$ [8]) even with relatively heavy squarks ($m_{\tilde{q}}\sim 1\,\mathrm{TeV}$), and can be even comparable to $\epsilon_K^{(\mathrm{exp})}$. Of course, if M_{ν_R} is smaller, or if there is an accidental cancellation among the phases, then the SUSY contribution to ϵ_K is more suppressed.

The information from ϵ_K plays a significant role in constraining the allowed region in the so-called ρ vs. η plane [28]. If the currently measured ϵ_K has a significant contamination of the SUSY contribution, however, ρ and η suggested by ϵ_K may become inconsistent with those from other constraints. Most importantly, the B

factories will measure the CP violation in the $B_d \to \psi K_S$ mode, and in the standard model, this process gives the phase $\arg(-[V_{\rm CKM}]_{cd}[V^*_{\rm CKM}]_{cb}/[V_{\rm CKM}]_{td}[V^*_{\rm CKM}]_{tb})$. As will be discussed below, the SUSY correction to this process is small at least in the parameter space we are discussing. By comparing ρ and η suggested by ϵ_K and those from $B_d \to \psi K_S$, we may see an anomaly arising from the SUSY loop effect if we fit the data just assuming the standard model.

Other check point is the K_L-K_S mass difference

$$\Delta m_K = \frac{|\langle K|\mathcal{H}_{\rm eff}|\bar{K}\rangle|}{m_K}, \tag{22}$$

since, in some case, SUSY contribution to Δm_K is significantly large. In our case, however, Δm_K is less important than the ϵ_K parameter; with $\Delta^{(R)}_{12} \sim \Delta^{(L)}_{12} \sim O(10^{-4} \div 10^{-3})$, the SUSY contribution to the Δm_K parameter is $O(1\%)$ level of its experimental value.

Finally, we discuss the contributions to $\Delta B = 2$ processes. Since the $\Delta^{(R,L)}_{13}$ and $\Delta^{(R,L)}_{23}$ parameters are non-vanishing, they may change the standard model predictions to the B decays. In particular, due to the possible GUT phases $\phi^{(Q)}$ and $\phi^{(L)}$, the CP violations in B systems as well as the mass differences Δm_B may be affected. $\Delta^{(R,L)}_{13}$ affects the B_d meson. With $\Delta^{(R)}_{13} \lesssim O(10^{-3})$ and $\Delta^{(L)}_{13} \sim O(10^{-4})$, however, SUSY contribution to Δm_{B_d} is also small, and is at most $\sim 1\%$ [22]. This also means that the phase in the B_d-$\bar{B}_d$ mixing matrix is dominated by the standard model contribution.

Because of the large 2-3 mixing in the V_L matrix, the SUSY contribution to the Δm_{B_s} parameter is more enhanced. In the standard model, the ratio $\Delta m_{B_s}/\Delta m_{B_d}$ is approximately $|[V_{\rm CKM}]_{ts}/[V_{\rm CKM}]_{td}|^2$. With this relation, we found that the SUSY contribution to Δm_{B_s} can be as large as $\sim 10\%$ of the standard model contribution when y_{ν_3} is maximally large. This fact also suggests that a sizable correction may be possible to the phase in the B_s-$\bar{B}_s$ mixing matrix because $\Delta^{(R)}_{23}$ has a phase equal to $e^{-i(\phi^{(L)}_2 - \phi^{(L)}_3)}$. Very small CP asymmetry is expected in the decay modes like $B_s \to \phi\eta$, $D_s\bar{D}_s$, and $\phi\eta'$ in the standard model, and the new source of the CP violation may affect the standard model predictions. Notice that, in the model without the right-handed neutrinos, $\Delta^{(L)}_{23}$ is approximately proportional to $[V^*_{\rm CKM}]_{ts}[V_{\rm CKM}]_{tb}$, while $\Delta^{(R)}_{23}$ is negligibly small. Then, the SUSY contribution to the matrix element $\langle B_s|\mathcal{H}_{\rm eff}|\bar{B}_s\rangle$ has almost the same phase as the standard model contribution, and hence the phase in the B_s-$\bar{B}_s$ mixing matrix is not significantly affected.

In summary, in the SUSY SU(5) model with the right-handed neutrinos, sizable CP and flavor violations are possible in the $\tilde{d}_R$ sector, which is not the case in models without the right-handed neutrinos. Importantly, with such effects, the SUSY contribution to the ϵ_K parameter can be as large as the experimental value ($\epsilon^{(\rm exp)}_K \simeq 2.3 \times 10^{-3}$) if the right-handed neutrino mass M_{ν_R} is high enough. In this paper, we only considered the $\Delta S = 2$ and $\Delta B = 2$ processes. In the future, however, other

CP violation informations will be available, in particular, in $\Delta S = 1$ and $\Delta B = 1$ processes. Given the fact that there can be extra sources of the CP violation, it would be desirable to measure as much CP violations as possible and to test standard model predictions.

Note added: after the completion of the main part of this work, the author was noticed a paper by S. Baek, T. Goto, Y. Okada and K. Okumura [29] which discussed implications of the right-handed neutrinos to the B physics in the SUSY GUT, in particular, to the Δm_{B_s} parameter. In their paper, however, effects of the GUT phases $\phi^{(Q)}$ and $\phi^{(L)}$ are not discussed.

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